Turbulent Liquid Flow Down Vertical Walls

H. H. BELKIN, A. A. MacLEOD, C. C. MONRAD, and R. R. ROTHFUS

Carnegie Institute of Technology, Pittsburgh, Pennsylvania

Photographic methods have been used to study the behavior of water flowing freely down vertical surfaces under the influence of gravity at Reynolds numbers between 200 and 30,000. The physical appearances of the liquid layers in transitional and fully turbulent flow have been noted. Layer thicknesses have been obtained from high-speed photographs and correlated with liquid Reynolds numbers, the range of experimental data thus being extended into the fully turbulent region. A simple basis of comparison with flow between parallel plates has been developed.

The free flow of liquids down vertical walls has been studied by a number of investigators. Almost all the experimental data on liquid depth however have been taken in the region of viscous flow. It is the purpose of this paper to report data which extend thickness measurements into the fully turbulent range. The present experiments utilize water at ordinary room temperature and cover a range of liquid-phase Reynolds numbers from 200 to 30,000.

Kirkbride measured the thicknesses of falling liquid films by means of direct contact with a micrometer (*8*). Fallah, Hunter, and Nash obtained average thicknesses from measurements of liquid holdup on vertical walls (*3*). Friedman and Miller also used a drainage method (*4*). Grimley measured average layer thicknesses by means of the electrical resistance of the liquid and checked his results through an optical shadow method (*5*). Dukler and Bergelin used a technique involving electrical capacitance and reached an upper Reynolds number of 3,000 (*2*). Chew used a photometric method to determine thicknesses for flow down mirrored glass plates (*1*). Kamei and his associates used a modified holdup technique and obtained a few points in the lower turbulent range as well as many in the viscous range (*7*). Jackson measured thicknesses by means of a radioactive tracer method and reported results for Reynolds numbers up to 5,600 (*6*). In total some sixteen investigators using various experimental methods have reported 504 thickness measurements, almost all of which lie in the Reynolds number range between 1 and 5,000. The present paper reports 85 additional points, mostly in the fully turbulent range.

Friedman and Miller injected a dye on the surface of a falling liquid layer and timed its descent to obtain the surface velocity. Grimley measured the velocities of colloidal particles suspended in falling liquid layers by means of a traveling microscope. In both cases the investigators concluded that the surface velocity exceeds its theoretical value over most of the viscous flow range. This behavior seems to be associated with surface waves which have their inception at very low Reynolds numbers. Jackson has shown that waves first appear near the top of the vertical surface when the velocity of wave generation equals the average velocity of the liquid layer, or in other words when the Froude number is unity. The corresponding Reynolds number has been observed to be in the vicinity of 12 to 25. The waves persist at higher Reynolds numbers and pass through a point of maximum amplitude at a Reynolds number of 100 to 200. Grimley has observed that the amplitude then decreases slowly until an almost smooth surface is again attained at a Reynolds number of about 1,000. Regardless of these complications however observers generally agree that the liquid-layer thickness approximates the theoretical relationship for flat-plate flow throughout the viscous or pseudoviscous flow range.

The classical expression for the liquid-layer thickness in fully viscous flow was developed by Nusselt (*9*) on the basis of several assumptions. A liquid of constant viscosity and density was assumed to flow steadily down a flat, vertical surface at the rate of Γ mass units per unit time and unit breadth of the surface. The shearing force at the interface between the liquid and its bounding gas was taken to be zero, and the density of the gas was assumed negligible. Nusselt's equation for the layer thickness can be written in the form

$$y_b g^{\frac{1}{3}} \rho^{\frac{2}{3}} \mu^{-\frac{2}{3}} = 0.909 \left(\frac{4\Gamma}{\mu}\right)^{1/3} \tag{1}$$

where $4\Gamma/\mu$ is the bulk Reynolds number of the liquid layer, and $y_b g^{1/3} \rho^{2/3} \mu^{-2/3}$ or $y_b g^{1/3} \nu^{-2/3}$ is a dimensionless thickness parameter. Although strictly speaking Equation (1) applies only to flow down a flat surface, the layer thickness is normally small enough for any curvature of the surface to be neglected. Similar equations can be developed for cases in which the curvature must be considered.

Dukler and Bergelin have presented an expression for the liquid-layer thickness in fully turbulent flow based on the generalized velocity distribution for smooth tubes. Reasonably good agreement with experimental data was demonstrated in the very low turbulent range. Extensions of essentially the same method to cases involving shearing forces at the gas-liquid interface in condensers have recently been presented by Rohsenow and his associates (*10*).

Conflicting effects of surface tension have been reported by several investigators. The influence of surface waves on layer thickness, the mechanism of their formation and movement, and the changes in local velocities occasioned by their passing are not well understood. Jackson

---

H. M. Belkin is at present with the Creole Petroleum Corporation, Caracas, Venezuela, and A. A. MacLeod is with E. I. duPont de Nemours and Company, Inc., Wilmington, Delaware.

The original data and additional photographs appear in two theses available on interlibrary loan from Carnegie Institute of Technology, Pittsburgh 13, Pennsylvania. The silhouette method is described in the D.Sc. thesis by A. A. MacLeod; the direct lighting method is presented in the Ph.D. thesis by H. M. Belkin.

however has offered a qualitative description of wave motion and a quantitative treatment of the relationship between surface velocity and the Froude number.

EXPERIMENTAL METHOD

The data reported in this paper were obtained in two pieces of apparatus designed along similar lines. In both cases distilled water was made to fall freely down the outer surface of a smooth, vertical rod after being extruded from an annular orifice at the top of the test section. The rod diameters were 0.934 and 0.935 in. In both pieces of equipment several orifices having different clearances were used, depending on the flow rate of the liquid layer. In both cases the water was recycled, and its temperature was maintained within 0.1°C. of the room temperature by partial bypassing through a cooler. Flow rates were measured by rotameters. The test sections were mounted in heavy frameworks to minimize vibrations, and the overhead supply tanks were well baffled to dampen pulsations of the flow.

The thickness of the water layer was determined at various flow rates by comparing high-speed photographs of the liquid with pictures of the dry rod. In one apparatus the illumination from a 10,000-volt spark of 5 μsec. duration produced a silhouette of the wetted rod. In the other apparatus the light of a 10,000-volt, 50-μsec. spark directly illuminated the water layer so that qualitative information about its physical appearance might be obtained in addition to the thickness data. A spark of short duration was necessary to eliminate blurring of the photographs owing to rapid local movements in the liquid layer. The cameras were centered on a point about 3 ft. from the annular entrance in most of the runs, although checks were made at other levels as well. Three pictures were obtained at each flow rate. No attempt was made to measure the effect of the falling liquid layer on the otherwise quiet room air surrounding the rods.

The meaning of the term *liquid layer thickness* varies somewhat in the published literature, depending on the particular method of measurement being reported. In the present case the meaning can be indicated by developing the working equation. If a side-view or elevation photograph of the dry rod and a corresponding photograph of the wetted rod are carefully taken from the same position with the same camera and then enlarged to the same length, the diameter of the dry rod in the picture bears the same proportional relationship to the actual diameter of the dry rod as the average liquid-layer thickness in the picture does to the actual average liquid-layer thickness. The dry rod in the picture has a projected area equal to the product $L' \, d'$. Likewise the wetted rod in the picture has a projected area equal to the product $L' \, (d' + 2y_b')$. The difference in measured areas $(A_2 - A_1)$ is therefore simply $2y_b'L'$. Thus by

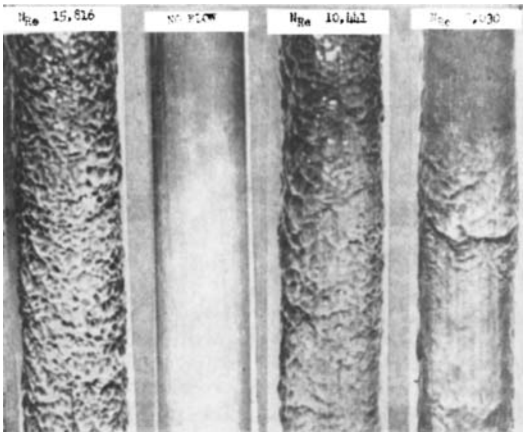

Fig. 1. Photographs of directly lighted vertical rod and falling liquid layers at several Reynolds numbers.

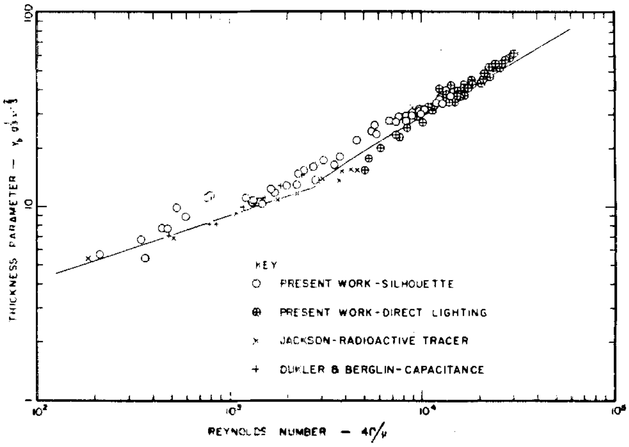

Fig. 2. Effect of Reynolds number on liquid-layer thickness with zero shear at the gas-liquid interface.

virtue of the proportional relationship originally cited it follows that

$$\frac{y_b}{d} = \frac{y_b'}{d'} = \frac{A_2 - A_1}{2L'} \frac{L'}{A'}$$

$$= \frac{A_2 - A_1}{2A_1} \qquad (2)$$

The actual diameter of the dry rod can be measured very accurately with a micrometer. It is apparent therefore that the accuracy and precision with which the thickness is determined must depend on the magnitudes of A_1 and A_2 as well as on the method of measurement. In the present work the enlarged images of the dry and wetted rod were traced on large sheets of paper, and the areas of the tracings were obtained by means of a polar planimeter. By this method the error in, for example, a 24-sq.-in. area was about 0.02 sq. in. when averaged over three pictures. Such an error might have resulted in an uncertainty of from 0.001 to 0.003 in. in the actual thickness, depending on the degree to which the original photograph was enlarged. It was noted however that a given operator tended to be consistently high or low in his readings. This acted strongly to reduce the actual error in the thickness computed by means of Equation (2).

The photographic method has the advantages of leaving the liquid undisturbed and being independent of elusive factors which influence the drainage type of measurements. On the other hand the photographs tend to indicate too high a value of the thickness because the troughs of the surface waves at the boundary of the image are sometimes masked by the crests of waves closer to the camera. It is possible for such an effect to make the reported thicknesses as much as 5 to 10% too high.

EXPERIMENTAL RESULTS AND DISCUSSION

Physical Appearance

Figure 1 shows typical photographs obtained with direct lighting at zero flow and three different Reynolds numbers in the lower turbulent range. There is evidence of discrete disturbance eddies at 5,030 Reynolds number. This suggests the persistence of transitional flow, since similar formations have been observed in tubes running full of liquid throughout the change from viscous to turbulent flow. On the other hand the appearance of the photographs at the higher Reynolds numbers suggests that full turbulence has been attained. Additional photographs not shown here indicate more intense turbulence but the same over-all type of flow at Reynolds numbers between 16,000 and 30,000. The technique of direct lighting was used only at Reynolds numbers above 5,030.

There was no special significance in the fact that the silhouette method was used at lower flow rates than the technique of direct lighting. The center of attention on the Reynolds number scale just happened to be different at the particular time each method was utilized. The upper limit of the tests was fixed by the supply and discharge capacities of the equipment rather than by any sudden change in the behavior of the liquid layer, such as separation from the surface. The capacities were limited by factors associated with some contemplated diffusion studies in the same equipment.

Silhouettes were obtained down to a Reynolds number of about 200. It was observed that ripples in the liquid layer increased in number but decreased in amplitude as the Reynolds number was increased in the pseudoviscous range. At a Reynolds number of about 1,000 however heavier waves appeared. These increased in number and amplitude with further increase in Reynolds number, until at about 3,000 Reynolds number they covered the surface of the liquid completely. At still higher Reynolds numbers the waves continued to increase in amplitude and tended to move in groups.

In both cases the pictures showed that sinusoidal waves with their clearly defined troughs and crests, as determined by Grimley and Dukler and Bergelin, give way to a random distribution of shallow troughs and sharp crests in the fully turbulent region. No definite patterns of wave length or frequency could be established, but the turbulent waves had nearly the trochoidal form of waves generated by wind on large bodies of water. A trochoid is the curve generated by a point on a circle which rolls along the underside of a straight line. The pictures made it easy to visualize that a profile cutting across the indentation between adjacent crests might closely resemble such a form.

Liquid-Layer Thickness

Figure 2 summarizes the experimental results of the thickness determinations. The points selectively tabulated by Dukler and Bergelin and by Jackson in their respective papers are also included for the purposes of comparison.

It is apparent that the data do not make it possible to fix the exact Reynolds number at which the advent of turbulence begins to affect the thickness parameter. The present data obtained by the silhouette technique approximate those of Dukler and Bergelin. On the other hand the results of the direct-lighting technique are in close agreement with Jackson's data. It cannot be said whether the differences are due to experimental errors or are caused by actual alterations in the stable flow at a given Reynolds number.

Some factors in the design and performance of the equipment are relevant to the discussion of critical Reynolds number. The liquid was put on the dry rod by means of annular orifices. The outer portions of the liquid layer were therefore accelerated after application in any case, but the over-all acceleration depended on how closely the orifice clearance matched the final average thickness of the freely falling liquid. Gross accelerations appeared to be confined to the top 6 to 12 in. of the test section. The photographs of the liquid layer were normally taken 36 to 40 in. from the top. In addition a few pictures were taken at 62 in. and analyzed in the usual way. Coincident linear variations of thickness with mass flow rate through a given orifice were obtained at both distances. Thus the values of the thickness parameter shown in Figure 2 should apply at all points on the vertical surface below the region of gross acceleration.

On the other hand it was observed that when the liquid level in the supply tank directly above the test section was permitted to become low, changes occurred in the liquid layer. The surface appeared to have additional wave fronts superimposed on the smaller waves in the falling layer. Furthermore the liquid thickness was found to be greater at low tank levels than might be predicted from data obtained at higher levels with the same orifice. Consequently the tank levels were held at a more or less constant height by utilizing several orifice sizes over the desired range of flow rates. The Reynolds number region covered by a given orifice was permitted to overlap its neighbors in order to check the reproducibility of the liquid-layer thicknesses. No appreciable effect of changing orifice sizes was evident. Such information is merely comparative however and does not necessarily mean that the measured thicknesses were entirely free of tank-level effects or persistent entrance disturbances. Therefore the spread of the data in Figure 2 occurs for reasons that cannot be pinpointed by means of the present experimental data. Other investigators however seem to have encountered the same order of uncertainty in fixing the critical Reynolds number on the basis of thickness measurements.

Correlation of Thickness Data

The thickness parameter ($y_b g^{1/3} \nu^{-2/3}$) is related to the Reynolds number ($4\Gamma/\mu$) as shown in Figure 2. Examination of the viscous or pseudoviscous region leads to the conclusion, expressed by numerous investigators, that the experimental data closely follow Equation(1). In other words the thickness of the liquid layer can be predicted with sufficient accuracy by treating the fluid as if it were flowing in viscous motion between two parallel flat plates separated by a clearance equal to twice the thickness of the actual falling layer. The data indicate that such thicknesses occur regardless of the many complicating factors introduced by the presence of a free surface at the gas-liquid interface.

It might reasonably be postulated therefore that transitional and fully turbulent liquid layers can also be handled with sufficient accuracy in the manner of flow between parallel plates. While such an approach is neither particularly new nor different, some recent friction data for parallel plates reported by Walker, Whan, and Rothfus (*11*) permit a straightforward analysis to be made.

If the curvature of the vertical surface and the shearing force at the gas-liquid interface can both be neglected, the skin friction at the solid surface must be

$$\tau_0 g_c = \rho g y_b \tag{3}$$

by virtue of a simple force balance on the liquid. Therefore the Fanning friction factor can be written

$$f = \frac{2\tau_0 g_c}{\rho V^2} = \frac{2 g y_b}{V^2} \tag{4}$$

The Reynolds number for flow between parallel plates is commonly taken to be

$$N_{Re} = \frac{4 y_b V}{\nu} \tag{5}$$

and so through the combination of Equations (4) and (5)

$$N_{Re}\sqrt{f} = \frac{4\sqrt{2}\, y_b^{\frac{2}{3}} g^{\frac{1}{2}}}{\nu} \tag{6}$$

or

$$y_b g^{\frac{1}{3}} \nu^{-\frac{2}{3}} = 0.315 (N_{Re}\sqrt{f})^{\frac{2}{3}} \tag{7}$$

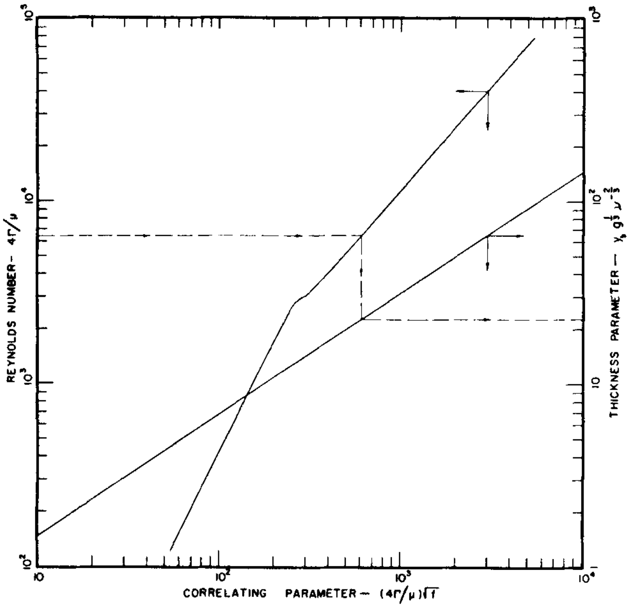

Fig. 3. Relationship between the Reynolds number and liquid-layer thickness predicted from parallel-plate friction data.

Since Equation (7) is developed solely from a force balance and definitions, it is independent of the stable type of flow in the liquid layer. The success with which it can be used to predict liquid-layer thickness at a given Reynolds number depends on the accuracy of two assumptions. First it is assumed that the liquid behaves as it would if the gas-liquid interface were not present; that is, the thickness of the liquid layer remains independent of actual interfacial conditions. Second it is assumed that the actual liquid layer attains its critical Reynolds number at the same value as for flow between parallel plates and that the effect of laminar-turbulent transition on the friction factor is the same in both cases. If the assumptions are met satisfactorily, Equation (7) should be equally valid at all Reynolds numbers.

Accordingly the friction data of Walker, Whan, and Rothfus were used to prepare a logarithmic graph of the Reynolds number against the product $(N_{Re}\sqrt{\ })f$. Equation (7) was then plotted on the same set of coordinates as shown in Figure 3. The solid line in Figure 2 was constructed directly from the corresponding values of thickness parameter and Reynolds number appearing in Figure 3. It can be seen that the experimental data are well represented by the parallel-plate correlation. The critical Reynolds number of 2,700 obtained in the friction experiments appears to be a reasonably satisfactory value for the falling liquid layer as well.

It should be emphasized that Figure 2 does not prove the hydrodynamic equivalence of parallel-plate flow and free flow down a vertical surface. In fact, it implies nothing about the mechanics of the liquid motion but simply indicates that the parallel-plate relationship seems to be a satisfactory basis for interpolating and extrapolating liquid-layer thicknesses over a long range of Reynolds numbers.

It is interesting to note the relationship between the Fanning friction factor for parallel plates and the Froude number of the liquid layer. Jackson used the Froude number in the form

$$N_{Fr} = V/(gy_b)^{\frac{1}{2}} \tag{8}$$

as a criterion for wave inception. When Equations (4) and (8) are combined, the Froude number can be expressed in terms of the Fanning friction factor as

$$N_{Fr} = \sqrt{2/f} \tag{9}$$

Consequently Equation (7) can equally well be written in the form

$$y_b g^{\frac{1}{3}} \nu^{-\frac{2}{3}} = 0.397(N_{Re}/N_{Fr})^{\frac{2}{3}} \tag{10}$$

Once again the last equation is independent of the type of flow existing in the liquid layer, since it is simply another form of Equation (7).

The Reynolds number and Froude number are related through Equation (9) in the simple case under consideration. Therefore when the Froude number is equal to unity, the condition for wave inception according to Jackson, the Fanning friction factor must be 2.0. For viscous flow between parallel plates, $f = 24/N_{Re}$, and so the Reynolds number at wave inception is 12. As might be expected, this result is in close agreement with Jackson's observations in the region very near the top of his test section.

NOTATION

A_1 = projected area of dry rod in picture, sq. ft.
A_2 = projected area of wetted rod in picture, sq. ft.
d = actual diameter of dry rod, ft.
d' = diameter of dry rod in picture, ft.
f = Fanning friction factor = $\dfrac{2T_0 gc}{\rho V^2}$ dimensionless
g = acceleration due to gravity, ft./sec.²
g_c = conversion factor in Newton's second law of motion = 32.2 (lb.-mass)(ft.)/(lb.-force)(sec.²)
L' = length of picture, ft.
N_{Fr} = Froude number = $V/(gy_b)^{1/2}$, dimensionless
N_{Re} = Reynolds number = $4\Gamma/\mu$, dimensionless
V = bulk average linear fluid velocity, ft./sec.
y_b = actual thickness of falling liquid layer, ft.
y_b' = liquid layer thickness in picture, ft.

Greek Letters

Γ = mass flow rate of liquid per unit breadth of vertical surface, lb.-mass/(sec.)(ft.)
μ = viscosity of liquid, lb.-mass/(sec.)(ft.)
ρ = density of liquid, lb.-mass/cu. ft.
τ_0 = skin friction at solid surface, lb.-force/sq. ft.
ν = kinematic viscosity of liquid, sq. ft./sec.

LITERATURE CITED

1. Chew J. N., Ph.D. thesis, Univ. Texas, Austin (1953).
2. Dukler, A. E., and O. P. Bergelin, *Chem. Eng. Progr.*, **48**, 557 (1952).
3. Fallah, R., T. G. Hunter, and A. W. Nash, *J. Soc. Chem. Ind. (London)*, **53**, 369T (1934).
4. Friedman, S. J., and C. O. Miller, *Ind. Eng. Chem.*, **33**, 885 (1941).
5. Grimley, S. S., *Trans. Inst. Chem. Eng. (London)*, **23**, 228 (1945).
6. Jackson, M. L., *A.I.Ch.E. Journal*, **1**, 231 (1955).
7. Kamei, S., J. Oishi, H. Iijima, M. Kawamura, and M. Itai, *Chem. Eng. (Japan)*, **18**, 545, (1954).
8. Kirkbride, C. G., *Trans. Am. Inst. Chem. Engrs.*, **30**, 170 (1933).
9. Nusselt, W., *Z. Ver. deut. Ing.*, **60**, 541, 569 (1916).
10. Rohsenow, W. M., J. H. Webber, and A. T. Ling, *Trans. Am. Soc. Mech. Engrs.*, **78**, 1637 (1956).
11. Walker, J. E., G. L. Whan, and R. R. Rothfus, *A.I.Ch.E. Journal*, **3**, 484 (1957).

Manuscript received May 5, 1958; revision received August 4, 1958; paper accepted May 6, 1958. Paper presented at A.I.Ch.E. Salt Lake City meeting.